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# CONNECTED AND TOTAL EDGE DOMINATION IN BOOLEAN FUNCTION GRAPH B (G, L(G), NINC) OF A GRAPH

**S. Muthammai[1] and S. Dhanalakshmi[2]**
*AlagappaGovernment Arts College, Karaikudi[1].*
*Government Arts College for Women(Autonomous), Pudukkottai.[2]*

**Abstract**

For any graph G, let V(G) and E(G) denote the vertex set and edge set of G respectively. The Boolean function graph B(G, L(G), NINC) of G is a graph with vertex set V(G)$\cup$E(G) and two vertices in B(G, L(G), NINC) are adjacent if and only if they correspond to two adjacent vertices of G, two adjacent edges of G or to a vertex and an edge not incident to it in G. For brevity, this graph is denoted by $B_1$(G). In this paper, Connected edge domination and total edge domination numbers of Boolean Function Graph B(G, L(G), NINC) of some standard graphs are obtained.

**Keywords:** Boolean Function graph, Edge Domination Number

## 1. INTRODUCTION

Graphs discussed in this paper are undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. A subset D of V is called a dominating set of G, if every vertex not in D is adjacent to some vertex in D. The domination number $\gamma$(G) of G is the minimum cardinality taken over all dominating sets of G. An edge e of a graph is said to be incident with the vertex v if v is an end vertex of e. In this case, it can also be said that v is incident with e.

A subset F $\subseteq$ E is called an edge dominating set of G, if every edge not in F is adjacent to some edge in F. The edge domination number $\gamma'$(G) of G is the minimum cardinality taken over all edge dominating sets of G. An edge dominating set X of G is called a total edge dominating of G if the induced subgraph $\langle X\rangle$has no isolated edges.

The total edge domination number$\gamma_t'$(G) of G is the minimum cardinality taken over all of total edge dominating sets of G. An edge dominating set X of is called a connected edge dominating sets of G, if the induced subgraph $\langle X\rangle$is connected. The connected edge domination number $\gamma_c'$(G) of G is the minimum cardinality taken over all connected edge dominating sets of G. The concept of edge domination was introduced by Mitchell and Hedetniemi [6]. Arumugam and Velammal [1] have discussed edge domination number and edge domatic number. Vaidya and Pandit [7] determined edge domination number of middle graphs, total graphs and shadow graphs of $P_n$ and $C_n$. For graph theoretic notations and terminology, Harary [2] is followed.

For a real x, $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

**Theorem 1.1. [6]** For any (p, q) graph G, $\gamma' \leq \lfloor p/2 \rfloor$

**Therorem 1.2. [3]** G and L(G) are induced subgraphs of $B_1(G)$

**Theorem 1.3.[3]** Number of vertices in $B_1(G)$ is p+q and if $d_i = deg_G(v_i)$, $v_i \in V(G)$, then the number of edges is $B_1(G)$ is $q(p-2)+ ½\sum_{1\leq i\leq p} d_i^2$.

**Theorem 1.4.[3]**The degree of a vertex of G in $B_1(G)$ is q and the degree of a vertex e′ of L(G) in $B_1(G)$ is $deg_{L(G)}(e') + p - 2$. Also if $d^*(e')$ is the degree of a vertex e′of L(G) in $B_1(G)$, then $0 \leq d^*(e') \leq p+q-3$. The lower bound is attained, if $G \cong K_2$ and the upper bound is attained, if $G \cong K_{1,n}$ for $n \geq 2$.

**Theorem 1.5. [3]** $B_1(G)$ is disconnected if and only if G is one of the following graphs: $nK_1$, $K_2$, $2K_2$ and $K_2 \cup nK_1$, for $n \geq 1$.

In this paper, connected edge domination numbers of Boolean Function Graph B(G, L(G), NINC) of some standard graphs are obtained.

## 2. Connected edge domination in B(G, L(G), NINC) of a Graph

In the following connected edge domination number of $B_1(P_n)$, $B_1(C_n)$,$B_1(K_n)$ $B_1(K_{1,n})B_1(W_n)$ are found.

**Theorem 2.1.** For the Path $P_n$on vertices ($n \geq 4$),$\gamma_c'$ $(B_1(P_n))$ =2n-3

**Proof:** Let $v_1, v_2,...,v_n$ and $e_{12}, e_{23}, …, e_{n-1,n}$ be the vertices and edges of $P_n$ respectively. Then $v_1, v_2, ..., v_n, e_{12}, e_{23}, …e_{n-1,n} \in V((B_1(Pn))$ where$e_{i,i+1} = (v_i, v_{i+1})$, i= 1, 2, …, n -1.$B_1(Pn)$ has 2n-1 vertices and $n^2 – n-1$ edges.

Let $F_m$ ={$(v_i,v_{jk})$ / 1 ≤i≤ n, j≡ (i+m) (mod (n-1)),k ≡ i+(m+1)(mod(n-1))} and
F =$(\bigcup_{m=1}^{n-2} Fm) \cup \{(v_1, e_{n-1}), (v_n,e_{n-2,n-1})\}$. Then $E(B_1(P_n)) = E(P_n) \cup E(P_{n-1}) \cup F$.
If D ′= $\{\bigcup_{i=1}^{n-2}(v_i,e_{i+1,i+2}),(e_{i,i+1},e_{i+1,i+2})\} \cup \{(v_{n-1}, e_{12})\}$, then $D' \subseteq E(B_1(P_n))$. D′ dominates edges of $P_n$,$P_{n-1}$ andF. D′ is anedge dominating set of $B_1(Pn)$. Also,$\langle D'\rangle \cong P_{n-1}^+$.
Therefore, D′ is a connected edge dominating set of $B_1(Pn)$ and hence
$\gamma_c'$ $(B_1(P_n))$≤|D′|=2(n-2)+1=2n-3.Let D′′ be a minimum edge dominating set of $B_1(Pn)$ .To dominate the edges of $B_1(Pn)$. D′′contains atleast (n-2)edges of L(Pn) and hence
|D′|≥ n-2+n-1 = 2n-3.Therefore, $\gamma_c'$ $(B_1(P_n))$ =2n-3.

**Remark:2.1**$\gamma_c'$ $(B_1(P_3))$ = 3

**Theorem:2.2.** For the Cycle $C_n$on n vertices ($n \geq 5$) vertices, $\gamma_c'$ $(B_1(C_n))$ =2n-3.
**Proof:** Let $v_1, v_2, ...,v_n$ be the vertices and $e_{12}, e_{23}, …, e_{n-1,n},e_{n1}$ are the edges of $B_1(C_n)$ where
$e_{i,i+1} = (v_i,v_{i+1})$, i = 1, 2, …, n – 1, $e_{n1}= (v_n, v_1)$. $B_1(C_n)$ has 2n vertices and $n^2$ edges.
Let $F_m$= {$(v_i, e_{jk})$ /1 ≤ i ≤ n, j ≡ (i+m)(mod n), k≡ (i+(m+1))(mod n), $e_{01} = e_{n1}$} and F =$\bigcup_{m=1}^{n-2} Fm$
$B_1(C_n) = E(2C_n) \cup F$.$|E(B_1(C_n))|$=2n +n(n-2) = $n^2$. Let D′ = $\bigcup_{i=1}^{n-2}\{(v_i, e_{i+1,i+2}), (e_{i,i+1}, e_{i+1,i+2})\} \cup \{(v_{n-1},e_{12})\}$. Then D′ is a edge dominating set of $B_1(C_n)$.
Also $\langle D'\rangle \cong P_{n-1}^+$. Therefore ,D′ is a connected edge dominating set of $B_1(C_n)$.

$\gamma_c'(B_1(C_n)) \leq |D'| = 2(n-2) + 1 = 2n - 3$. Let D'' be a minimum connected edge dominating set of $B_1(C_n)$. D'' contains atleast (n-1) edges of F and (n-2) edges of $L(C_n)$. $|D''| \geq 2n-3$. Therefore, $\gamma_c'(B_1(C_n)) = 2n-3$.

**Remark: 2.2**

(i) $\gamma_c'(B_1(C_3)) = 5$

(ii) $\gamma_c'(B_1(C_4)) = 6$

**Theorem:2.3.** For the complete graph $K_n$ on n (n≥5) vertices, $\gamma_c'(B_1(K_n)) = (n+3)(n-2)/2$.

**Proof:** Let $v_1, v_2, \ldots, v_n$ be the vertices of $K_n$ and $E(K_n) = \{e_{ij} = (v_i, v_j) / 1 \leq i \leq n, 1 \leq j \leq n, i \neq j\}$ $B_1(K_n)$ has $n(n+1)/2$ vertices. $|E(B_1(K_n))| = |E(K_n)| + |E(L(K_n))| + n(n-1)(n-2)/2 = n(n-1)(2n-3)/2$. Let $F_1 = \cup_{j=3}^{n}\{(v_1\, e_{2j})\}, F_2 = \cup_{j=4}^{n}\{(v_{2,} e_{3j})\}$, $F_3 = \cup_{j=5}^{n}\{(v_{3,} e_{4j})\}$
. …. $F_{n-3} = \cup_{j=n-3}^{n}\{(v_{n-3,} e_{n-2,j})\}$, $F_{n-2} = \cup_{j=2, j\neq n-2}^{n}\{(v_{n-2,} e_{1j})\}$, $F_{n-1} = \cup_{j=1}^{n-1}\{(v_{i\,,} v_{i+1})\}$
and let $F = \cup_{i=1}^{n-1} F_i$. Then $F \subseteq E(B_1(Kn))$. F is a dominating set of $B_1(K_n)$. Let $P_n$ be the path induced by the vertices $v_{1,} v_{2, \ldots,} v_n$ . < F > is a graph obtained by attaching n-2, n-3, n-4, …, 2 and n-2 pendant edges at $v_{1,} v_{2, \ldots,} v_{n-3}, v_{n-2}$ of $P_n$ respectively. Therefore, F is a connected edge dominating set of $B_1(Kn)$ and hence, $\gamma_c'(B_1(K_n)) \leq |F| = |\cup_{i=1}^{n-1} F\, i\,| = (n-2) + (n-3) + \ldots + 2 + n-2 + n-1 = (n-1)n / 2 - 1 + n-2 = (n^2 - n - 2 + 2n - 4)/2 = (n^2 + n - 6)/2 = (n+3)(n-2)/2$. F is also a minimum connected edge dominating set of $B_1(K_n)$ and hence $\gamma_c'(B_1(K_n)) = (n+3)(n-2)/2$.

**Remark: 2.3**

(i) $\gamma_c'(B_1(K_3)) = 5$

(ii) $\gamma_c'(B_1(K_4)) = 6$

**Theorem:2.4.** For the star $K_1$,n on (n+1) vertices (n≥4), $\gamma_c'(B_1(K_{1,n})) = n+1$.

**Proof:** Let v, $v_1, v_2, \ldots, v_n$ be the vertices of $K_{1,n}$ with v as the central vertex. Let $e_i = (v, v_i)$, i =1, 2, 3, …, n be the edges of $K_{1,n.}$ Then $v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_n \in V((B_1(K_{1,n}))$. $B(K_1,n)$ has 2n+1 vertices and n(3n-1)/2 edges.

Let $D' = \{\cup_{i=1}^{n-1}(e_{i,} e_{i+1})\} \cup \{(v, v_1), (v_{1,} e_n)\}$ . Then $|D'| = n+1$. The edge $(v, v_1)$ in D' dominates all the edges of G and the edges $\cup_{i=1}^{n-1}(e_{i,} e_{i+1}), (v_{1,}\, e_n)$ dominate remaining edges of $K_{1,n}$ and <D'> $\cong P_{n+2.}$ Therefore, D' is a connected edge dominating set of $B_1(K_{1,}n)$ and hence
$\gamma'(B_1(K_{1,n}) \leq |D'| = n+1$. Let D'' be a connected edge dominating set of $k_{1,n}$. To dominate edges of $k_{1,n}$ ,D'' contains one edge of $k_{1,n,}$ and to dominate n(n-1) edges of the form $(v_{i,} e_j)$ ($e_j$ is not incident with $v_i$). D'' contains atleast (n-1) edges. Since <D''> is connected, D'' contains one more edge and hence $|D'| \geq n+1$. Therefore, $\gamma_c'(B_1(K_{1,n})) = n+1$.

**Therorem 2.5:** For the Wheel $W_n$ on n vertices (n ≥ 5), $\gamma_c'(B_1(W_n)) = 3n-5$.

**Proof:** Let $v_1, v_{2,} \ldots, v_n$ be the vertices of $W_n$ with $v_1$ as the central vertex and $e_{12}, e_{13}, \ldots, e_{1n}$, be the edges of $B_1(C_n)$ where $e_{1,\, i+1} = (v_i, v_{i+1})$, i = 2,3, …, n. Then $v_1, v_2, \ldots, v_n, e_{12}, e_{13}, \ldots, e_{1n,} e_{23}, \ldots, e_{n2} \in V((B_1(W_n))$. $B_1(W_n)$ has 2n-1 vertices and (n-1)(3n-4)/2 edges.

Let $F_1 = \cup_{i=1}^{n-1}\{(v_{i,}\ v_{i+1})\}$, $F_2 = \cup_{i=2}^{n-2}\{(v_{i,} e_{i+1,i+2})\}$

$F_3 = \bigcup_{i=2}^{n-2}\{(e_{i,i+1}, e_{1,i})\} \cup (e_{n-1,n}, e_{1n})$

Let D′ = $F_1 \cup F_2 \cup F_3$. $F_1$ and $F_2$ dominates all the edges of $W_n$ and edges of the form ( $v_i, e_{jk}$) where $e_{jk}$ is not incident with $v_i$. $F_2 \cup F_3$ dominates all the edges of L($W_n$). Therefore, D′ is a edge dominating set of $B_1(W_n)$. |D′| ≤ n-1+n-2+n-2 = 3n-5. <D′> is a graph obtained from $P_{n-2}^+$ by subdividing each pendant edge and then attaching a path of length 2 at a pendant vertex of $P_{n-2}$. D′ is a connected edge dominating set of $B_1(W_n)$.

Let D′′ be a minimum connected edge dominating set of $B_1(W_n)$. To dominate edges of $W_n$ and edges of the form ($v_i, e_{jk}$) and to maintain connectedness of <D′′>, D′′ contains atleast (n-1) edges of $W_n$, (n-2) edges of the form ($v_i, e_{jk}$) and(n-2) edges of L($W_n$).

Therefore, |D′| ≥ 3n-5. Hence,$\gamma_c'$ ($B_1(W_n)$) = 3n-5.

**Remark:2.4**Since every connected edge dominating set is also an edge dominating set of a graph G,$\gamma'$ ($B_1$(G)) ≤ $\gamma_c'$ ($B_1$(G))

**Remark: 2.5** Any connected edge dominating set is also a total edge dominating set and hence $\gamma_t'$ ($B_1$(G)) ≤ $\gamma_c'$ ($B_1$(G)).

## 3.Total edge dominationin B(G, L(G), NINC) of a Graph

In the following total edge domination number of $B_1(P_n)$, $B_1(C_n)$,$B_1(K_{1,n})$$B_1(W_n)$ are found.

**Theorem : 3.1** For thePath $P_n$ on n (n ≥ 4 ) vertices, $\gamma'_t(B_1(P_n)) \leq n$.

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices and $e_{i,i+1} = (v_i, v_{i+1})$ (i = 1, 2, …, n-1) be the edges of $P_n$. Then $v_1, v_2, \ldots, v_n$, $e_{12}, e_{23}, \ldots, e_{n-1,n} \in V(B_1(P_n))$. $B_1(P_n)$ has2n-1vertices and $n^2$ - n -1 edges.

Case (i): n is even

Let D′ = $\bigcup_{i=1}^{n/2}\{(v_{2i-1}, v_{2i})\}$ and D′′ = $\bigcup_{i=1}^{n-2/2}\{(v_{2i+1}, e_{2i-1,2i})\}$ and D = D′∪ D′′{ ( $v_i$, $e_{n-2,n-1}$)}

Then D ⊆ E ($B_1(P_n)$) and |D|= $\frac{n}{2}+\frac{n-2}{2}$+1 = n. D is an edge dominating set of $B_1(P_n)$ and < D >≅$\frac{n}{2}$ $P_3$ with central vertices $v_1, v_2, \ldots, v_{n-1}$ respectively.

Therefore, D is a total edge dominating set of $B_1(P_n)$ and hence $\gamma'_t(B_1(P_n)) \leq |D| = n$.

Case(ii): n is odd

Let F′ = $\bigcup_{i=1}^{n-1/2}\{(v_{2i-1}, v_{2i})\}$ and F′′ =$\bigcup_{i=1}^{n-3/2}\{(v_{2i+1}, e_{2i-1,2i})\}$

and let F = F′∪ F′′∪ {($v_{n-1}, v_n$) ( $v_i$, $e_{n-2,n-1}$)} then F ⊆ E ($B_1(P_n)$) and |F|= $\frac{n-1}{2}+\frac{n-3}{2}$+2 = n. F is an edge dominating set of $B_1(P_n)$ and < F >≅$\frac{n-3}{2}$ $P_3 \cup P_4$ where the central vertices of$P_3$ are$v_1, v_2, \ldots, v_{n-4}$and $P_4$ is induced by the edges ($v_{n-2}, v_{n-1}$), ($v_{n-1}, v_n$) and ($v_{n-2}, e_{n-4,n-3}$). Therefore, F is a total edge dominating set of $B_1(P_n)$ and hence $\gamma'_t(B_1(P_n)) \leq |F| = n$.

**Example:**

(1) Let $V(P_8) = \{ v_1, v_2, \dots , v_8\}$ and $E(P_8) = \{((v_i,v_{i+1})\ (i = 1, 2, \dots, 7)$.

Then $D = \{(v_1, v_2)\ (v_3, v_4)\ (v_5, v_6)\ (v_7, v_8)\ (v_1, e_{67})\ (v_3, e_{12})\ (v_5, e_{34})\ (v_7, e_{56})\}$ is an edge dominating set of $B_1(P_8)$ and $D \subseteq E(B_1(P_8))$ and $<D> \cong 4P_3$. D is a total edge dominating set of $B_1(P_8)$. Therefore, $\gamma'_t(B_1(P_8)) \leq 8$.

(2) Let $V(P_7) = \{ v_1, v_2, \dots , v_7\}$ and $E(P_7) = \{((v_i,v_{i+1})\ (i = 1, 2, \dots, 6)$.

Then $D = \{(v_1, v_2)\ (v_3, v_4)\ (v_5, v_6)\ (v_6, v_7)\ (v_1, e_{56})\ (v_3, e_{12})\ (v_5, e_{34})\}$is an edge domination set of $B_1(P_7)$ and $D \subseteq E(B_1(P_7)$ and $<D> \cong 2P_3 \cup P_4$ and D is a total edge dominating set of $B_1(P_7)$. Therefore, $\gamma'_t(B_1(P_7)) \leq 7$.

**Theorem:3.2** For the cycle $C_n$ on n( $n \geq 3$) vertices, $\gamma'_t(B_1(C_n)) \leq n$, if n is even

$\leq n+1$, if n is odd

Proof: Let $v_1, v_2, \dots , v_n$ be the vertices and $e_{i,i+1} = (v_i,v_{i+1})$ $(i = 1, 2, \dots, n-1)$ and $e_{n1} = (v_n, v_1)$ be the edges of $C_n$. Then $v_1, v_2, \dots ,v_n, e_{12}, e_{23},\dots, e_{n-1,n} e_{n1} \in V(B_1(C_n))$. $B_1(C_n)$ has 2n vertices and $n^2$ edges.

Case (i): n is even

Let $D' = \bigcup_{i=1}^{n/2}\{(v_{2i-1}, v_{2i})\}$ and $D'' = \bigcup_{i=1}^{(n-2)/2}\{(v_{2i+1}, e_{2i-1,2i})\}$ and $D = D' \cup D''\{ ( v_i, e_{n-2,n-1})\}$

Then $D \subseteq E(B_1(C_n))$ and $|D| = \frac{n}{2} + \frac{n-2}{2} + 1 = n$. D is an edge dominating set of $B_1(C_n)$ and $< D > \cong \frac{n}{2} P_3$ with central vertices $v_1, v_2, \dots , v_{n-1}$ respectively.

Therefore, D is a total edge dominating set of $B_1(C_n)$ and hence $\gamma'_t(B_1(C_n)) \leq |D| = n$.

Case(ii): n is odd

Let $F' = \bigcup_{i=1}^{n/2}\{(v_{2i-1}, v_{2i})\}$

$F'' = \bigcup_{i=1}^{(n-1)/2}\{(v_{2i+1}, e_{2i-1,2i})\}$ and let $F = F' \cup F'' \cup \{(v_1, e_{n-1, n})\}$ then $F \subseteq E(B_1(C_n))$ and $|F| = \frac{n}{2} + \frac{n-1}{2} + 1 = n+1$. D is an edge dominating set of $B_1(C_n)$ and $< D > \cong \frac{n-3}{2} P_3 \cup P_5$ where the central vertices of $P_3$ are $v_1, v_3, \dots , v_{n-4}$ and $P_5$ is induced by the edges $(v_{n-2}, v_{n-1})$, $(v_{n-1}, v_n)$ and $(v_n, e_{n-2,n-1})$ and $(v_{n-2}, e_{n-4,n-3})$. Therefore, D is a total edge dominating set of $B_1(C_n)$ and hence $\gamma' t(B_1(C_n)) \leq |D| = n+1$.

**Theorem: 3.3** For the star $K_{1,n}$ on (n+1) vertices ($n \geq 3$), $\gamma'_t(B_1(K_{1,n})) \leq n+1$

Proof: Let $v_1, v_2, v_3, \dots, v_{n+1}$ be the vertices of $K_{1,n}$, with $v_1$ as the central vertex. Let

$e_{1,i+1} = (v_1, v_i)$, $i = 2, 3, \dots, n+1$ be the edges of $K_{1,n}$. Then $v_1, v_2, \dots, v_{n+1}, e_{12}, e_{13} \dots, e_{1n+1} \in V(B_1(K_{1,n}))$. $B_1(K_{1,n})$ has 2n + 1 vertices and 2n+1 and (n(3n-1))/2 edges.

Case(i): n is odd

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Let $D' = \cup_{i=3}^{(n+3)/2}\{(v_i, e_{2i-2}), (e_{1,2i-2}, e_{1,2i-1})\}$ where $e_{1,n+2} = e_{12}$ and let $D = D' \cup \{(v_1, v_2), (v_2, e_{13})\}$ Then $D \subseteq E(B_1(K_{1,n}))$ and $|D| = 2[\frac{n+3}{2} - 2] + 2 = 2(\frac{n-1}{2}) + 2 = n+1$. D is an edge dominating set of $B_1(K_{1,n})$ and $\langle D\rangle \cong \frac{n+1}{2} p_3$ with central vertices $v_2, e_{14}, e_{16}, \dots, e_{1,n+1}$. Therefore D is a total edge dominating set of $B_1(K_{1,n})$ and hence $\gamma'_t(B_1(K_{1,n})) \leq |D| = n+1$.

case(ii): n is even

Let $F' = \cup_{i=3}^{n+2/2}\{(v_i, e_{1,2i-2}), (e_{1,2i-2}, e_{1,2i-1})\}$ and $F = F' \cup \{(v_1, v_2)\ (v_2, e_{13})\ (e_{1,n+1}, e_{12})\}$

$F \subseteq E(B_1(K_{1,n}))$ and $|F| = 2[\frac{n+2}{2} - 2] + 3 = n - 2 + 3 = n+1$. D is an edge dominating set of $B_1(K_{1,n})$ and $\langle D\rangle \cong \frac{n-2}{2} P_3 \cup P_4$, where the central vertices of $P_3$ are $v_2, e_{14}, e_{16}, \dots, e_{1,n-2}$ and the $P_4$ is induced by the edges $(v_{n-2}, e_{1n})\ (e_{1n}, e_{1,n+1})\ (e_{1,n+1}, e_{12})$. Therefore, D is a total edge dominating set of $B_1(K_{1,n})$ and hence $\gamma'_t(B_1(K_{1,n})) \leq |D| = n+1$.

**Theorem:3.4** For the Wheel $W_n (n \geq 5)$ on n vertices, $\gamma'_t(B_1(W_n)) \leq 2n - 2$

Proof:Let $v_1, v_2, v_3, \dots, v_n$ be the vertices of $W_n$ with $v_1$ as the central vertex. Let

$e_{1,i} = (v_1, v_i) (i = 2, 3, \dots, n)$ and $e_{i,i+1} = (v_1, v_{i+1})$ $(i = 2, 3, \dots, n-1)$ $e_{n2} = (v_n, v_2)$ be the edges of $W_n$. Then $v_1, v_2, \dots, v_n, e_{12}, e_{13} \dots, e_{1n}, e_{12}, e_{23}, \dots, e_{n-1,n} e_{n2} \in V(B_1(W_n))$. $B_1(Wn)$ has n+ n -1+n-1= 3n - 2 vertices and (n-1)(3n- 4))/2 edges.

Case(i): n is even

Let $D' = \cup_{i=3}^{n/2}\{(v_i, e_{1,2i-2}), (e_{1,2i-2}, e_{1,2i-1})\}$

$D'' = \cup_{i=2}^{\frac{n}{2}}\{\left(v_{\frac{n}{2}+i}, e_{2i-3,2i-2}\right)(e_{2i-3,2i-2}, e_{2i-2,2i-1})\}$ and let $D = D' \cup D'' \cup \{(v_1, v_2), (v_2, e_{13})\ (v_{n/2+1}, e_{1n}), (e_{1n}, e_{n2})\}$ then $D \subseteq E(B_1(W_n))$ and $|D| = 2(n/2 - 2) + (n/2 - 1) + 4 = 2n - 2$. D is a total edge dominating set of $B_1(W_n)$ and $\langle D\rangle \cong (n-1) P_3$ with central vertices $v_2, e_{14}, e_{16}, \dots, e_{1,n/2}, e_{12}, e_{34}, \dots, e_{n-3,n-2}$.

Case(ii): n is odd

Let $F' = \cup_{i=3}^{(n+1)/2}\{(v_i, e_{1,2i-2}), (e_{1,2i-2}, e_{1,2i-1})\}$

$F'' = \cup_{i=1}^{\frac{n-1}{2}}\{\left(v_{\frac{n+1}{2}+i}, e_{2i-1,2i}\right)(e_{2i-1,2i}, e_{2i,2i+1})\}$ and let $F = F' \cup F'' \cup \{(v_1, v_2), (v_2, e_{13})$ $F \subseteq E(B_1(W_n))$ and $|F| = 2(n+1/2 - 2) + 2(n-1/2) + 2 = 2n - 2$. F is a total edge dominating set of $B_1(W_n)$ and $\langle F\rangle \cong (n-1) P_3$. Therefore, F is a total edge dominating set of $B_1(W_n)$ and hence $\gamma'_t(B_1(W_n)) \leq |F| = 2n-2$.

**Theorem:3.5** G have a perfect matching, $\gamma'_t(B_1(G)) \leq 2\beta_1(G)$ if $\beta_1(G) > \alpha_0(L(G))$

$\leq 2\alpha_0(L(G))$ if $\beta_1(G) \leq \alpha_0(L(G))$

Proof: Let $K \subseteq E(G)$ be a perfect matching such that $|K| = k = \beta_1(G)$. Then $K \subseteq E(B_1(G))$.

Let K = $\{(v_1, u_1)\ (v_2, u_2)\ , \ldots, (v_k,u_k)\}$. Let M be a point cover of L(G) and let $|M| = \alpha_0(L(G)) = m = \{e_1, e_2 \ldots, e_m\}$
Case: (i) $k > m$ ( $\beta_1(G) > \alpha_0$)

Choose one of $u_i$ and $v_i$. Let it be $v_i$ (i= 1, 2, … ,k). Choose a distinct vertex $e_i$ in M such that the corresponding edge in G is not incident with $v_i$. Then the edge $(v_i,u_i) \in E(B_1(G))$. Let L be the set of all these edges. $|L| = k$. Then $L \subseteq E(B_1(G))$. Let $D = K \cup L \subseteq E(B_1(G))$. K dominates all the edges of G in$B_1(G)$ and edges of the form (w, e) where $e \in E(G)$ is not incident with $w \in V(G)$. L dominates all the edges of L(G). Therefore, D is an edge dominating set of $B_1(G)$. Also $< D>$ contains no isolated edges. Therefore, D is a total edge dominating set of $B_1(G)$ and hence $\gamma'_t(B_1(G)) \leq |D| = |K \cup L| = 2K = 2\beta_1(G)$ .

Case(ii): $k \leq m$, that is $\beta_1(G) > \alpha_0(L(G))$. For each vertex $e_i \in M$, choose a vertex $u_i$(or) $v_i$, which is not incident with $e_i$. Then the edge $(v_i, e_i) \in E(B_1(G))$. Let N be the set of all these edges. $|N| = m$, $N \subseteq E(B_1(G))$. Then the set $D' = K \cup N$ is a total edge dominating set of $B_1(G)$ as in case(i). Therefore, $\gamma'_t(B_1(G)) \leq |D'| = |K \cup N| = \beta_1(G) + m = \beta_1(G) + \alpha_0(L(G) \leq \alpha_0(L(G))$.

Therefore, $\gamma'_t(B_1(G)) \leq 2\beta_1(G)$ if $\beta_1(G) > \alpha_0(L(G))$

$\leq 2\alpha_0(L(G))$ if $\beta_1(G) \leq \alpha_0(L(G))$.

## 4. CONCLUSION

In this paper, connected edge and total edge domination numbers of Boolean Function GraphB(G,L(G),NINC) of paths , cycles, complete graphs, stars, wheels are obtained.

## REFERENCE:

[1]. S Arumugam and S.Velammal, Edge domination in graphs ,Tairwanese Journal of Mathematics, Vol.2, pp.173- 179,1998.

[2]. Harary F, Graph Theory, Addison, – Wesley Reading Mass.,1969.

[3]. T. N. Janakiraman, S.Muthammai, M.Bhanumathi, On the Boolean Function Graph of a Graph and on its Complement,MathematicaBohemica, 130(2005), No.2, pp. 113-134.

[4]. T. N. Janakiraman, S. Muthammai, M. Bhanumathi, Domination Numbers on the Boolean Function Graph of a Graph, Mathematica Bohemica, 130(2005), No.2, 135-151.

[5]. T.N.Janakiraman, S.Muthammai, M. Bhanumathi, Domination Numbers on the Complement of the Boolean Function Graph of a Graph, Mathematica Bohemica, 130(2005), No.3, pp. 247- 263.

[6]. S.Mitchell and S.T.Hedetniemi, Edge domination in trees, CongressusNumerantium, Vol. 19, pp. 489-509, 1977.

[7]. S. K. Vaidya and R.M. Pandit, Edge Domination in Some Path and Cycle Related Graphs, ISRN Discrete Mathematics, Vol. 2014, Article ID 975812, 5 pages.

[8]. S.Velammal, Equality of connected Edge Domination and Total Edge domination in Graphs, IERST, No:2319-7463, vol.3 Issue 5, May- 2014,pp.198-201.

[9]. S.Muthammai, S.Dhanalakshmi, Edge Domination in Boolean Function Graph B(G, L(G), NINC) of a Graph, IJIRSET Journal, Vol. 4, Issue 12,December 2015, pp.12346 – 12350.

[10]. S.Muthammai, S.Dhanalakshmi, Edge Domination in Boolean Function Graph B(G, L(G), NINC) of Corona of Some Standard Graphs, Global Journal of Pure and Applied Mathematics, Vol. 13, Issue 1, 2017, pp.152 – 155.